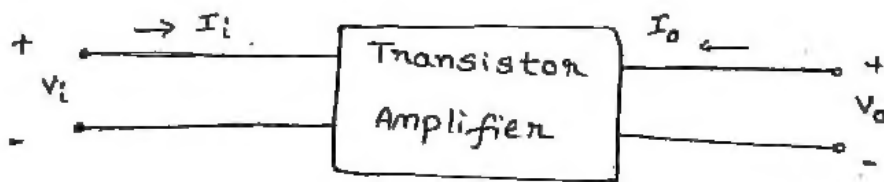


BJT AMPLIFIERS

H-Parameter Representation of a Transistor

A transistor can be treated as a two-port network



Hence I_i = Input current to the Amplifier

V_i = Input voltage to the Amplifier

I_o = output current of the Amplifier

V_o = output voltage of the Amplifier

Transistor is a current operated device.

Hence input voltage V_i and output current I_o are the dependent variables.

Input current I_i and output voltage V_o are Independent variables.

$$V_i = f_1(I_i, V_o)$$

$$I_o = f_2(I_i, V_o)$$

This can be written in the equation form as follows

$$V_i = h_{11} I_i + h_{12} V_o$$

$$I_o = h_{21} I_i + h_{22} V_o$$

The above equation can also be written using alphabetic notations

$$V_i = h_i I_i + h_r V_o$$

$$I_o = h_f I_i + h_o V_o$$

Definitions of h-Parameters:

The parameters in the above equation are defined as follows

$$h_{11} = h_i = \left. \frac{V_i}{I_i} \right|_{V_o=0} = \text{Input resistance with output short circuited.}$$

$$h_{12} = h_r = \left. \frac{V_i}{I_o} \right|_{I_i=0} = \text{Reverse voltage transfer ratio with input open circuited.}$$

$$h_{21} = h_f = \left. \frac{I_o}{I_i} \right|_{V_o=0} = \text{short circuit } \overset{\text{Forward}}{\text{current gain}} \text{ with output short circuited.}$$

$$h_{22} = h_o = \left. \frac{I_o}{V_o} \right|_{I_i=0} = \text{output Admittance with input open circuited.}$$

BJT H-parameter Model:

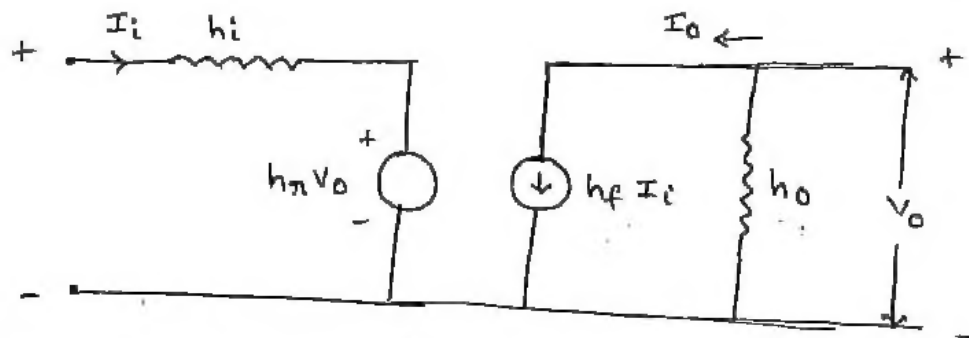
Based on the definition of hybrid parameters the mathematical model for two port networks known as h-parameter model (Hybrid parameter model) can be developed.

The two equations of a transistor is given by

$$V_i = h_i I_i + h_r V_o$$

$$I_o = h_f I_i + h_o V_o$$

Based on above two equations the equivalent circuit on Hybrid Model for transistor can be drawn.



Advantages (or) Benifits of h-parameters

- 1) Real numbers at audio frequencies
- 2) Easy to measure
- 3) can be obtained from the transistor static characteristic curves.
- 4) convenient to use in circuit analysis and design.
- 5) Easily convertable from one configuration to other
- 6) Most of the transistor manufacturers specify the h-parameters.

H parameter model for CE Configuration

Let us consider the common emitter configuration shown in figure below. The variables I_b , I_c , V_b and V_c represent total instantaneous currents and voltages.

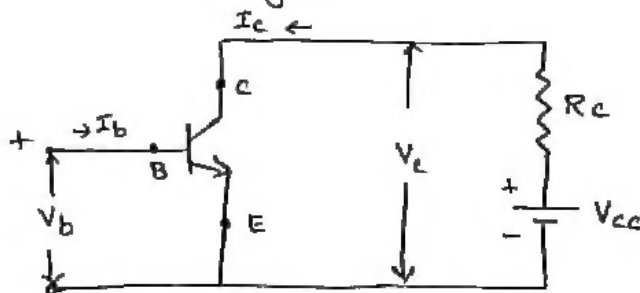


Fig: simple common emitter configuration

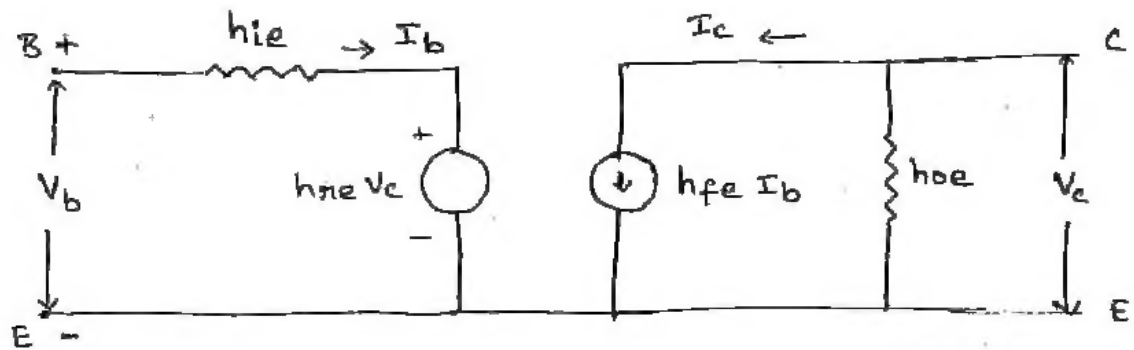
Here I_b - Input current

V_b - Input voltage

I_c - output current

V_c - output voltage

h-parameter model for common emitter configuration is shown in figure below.



$$V_b = h_{ie} I_b + h_{ne} V_c$$

$$I_c = h_{fe} I_b + h_{oe} V_c$$

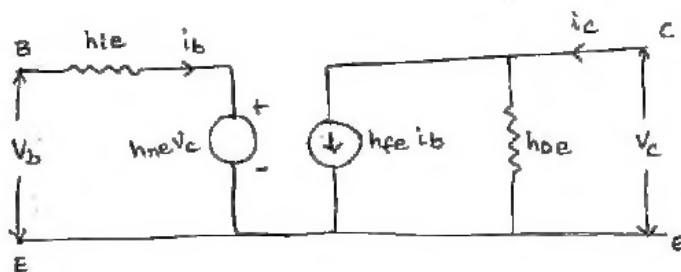
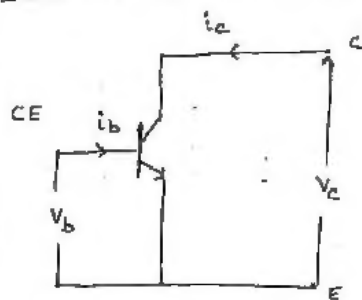
where
$$h_{ie} = \left. \frac{\Delta V_B}{\Delta I_B} \right|_{V_c = \text{constant}} = \left. \frac{V_b}{I_b} \right|_{V_c = \text{constant}}$$

$$h_{ne} = \left. \frac{\Delta V_B}{\Delta V_c} \right|_{I_B = \text{constant}} = \left. \frac{V_b}{V_c} \right|_{I_b = \text{constant}}$$

$$h_{fe} = \left. \frac{\Delta I_c}{\Delta I_B} \right|_{V_c = \text{constant}} = \left. \frac{i_c}{i_b} \right|_{V_c = \text{constant}}$$

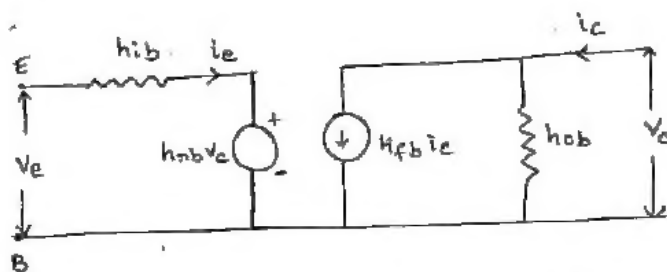
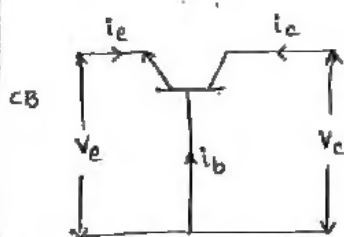
$$h_{oe} = \left. \frac{\Delta I_c}{\Delta V_c} \right|_{I_B = \text{constant}} = \left. \frac{i_c}{V_c} \right|_{I_b = \text{constant}}$$

Hybrid model for the transistor in three different configurations



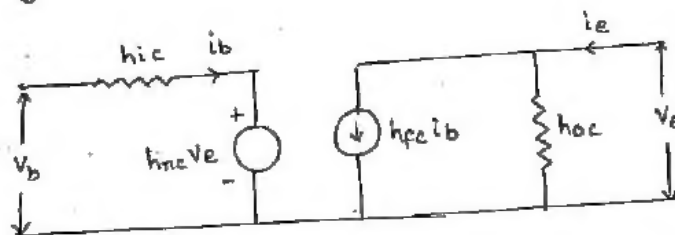
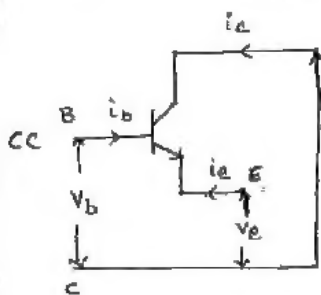
$$V_b = h_{ie} i_b + h_{ne} V_c$$

$$i_c = h_{fe} i_b + h_{oe} V_c$$



$$V_e = h_{ib} i_e + h_{nb} V_c$$

$$i_c = h_{fb} i_e + h_{ob} V_c$$



$$V_b = h_{ic} i_b + h_{nc} V_e$$

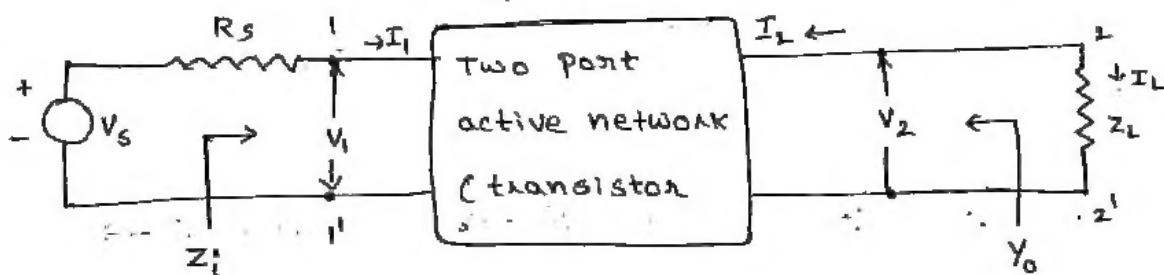
$$i_e = h_{fc} i_b + h_{oc} V_e$$

Typical h-parameter values for a transistor

Parameter	CE	CC	CB
h_i	1100Ω	1100Ω	22Ω
h_r	2.5×10^{-4}	1	3×10^{-4}
h_{fe}	50	-51	-0.98
h_o	$25\mu A/V$	$25\mu A/V$	$0.49\mu A/V$

Analysis of a transistor amplifier circuit using h-parameter model.

A transistor amplifier can be constructed by connecting an external load and signal source as indicated in figure below. and biasing the transistor properly.



The hybrid parameter model for above network is shown in figure below.

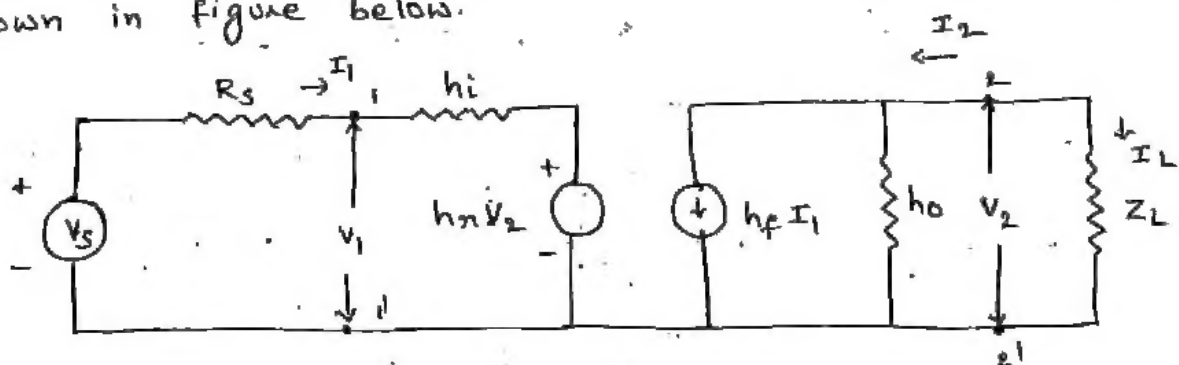


Fig: Transistor hybrid parameter model.

1) Current Gain (or) Current Amplification A_I :

For a transistor amplifier the current gain A_I is defined as the ratio of output current to input current.

$$A_I = \frac{I_L}{I_1} = \frac{-I_2}{I_1}$$

From the circuit $I_2 = h_f I_1 + h_o V_2 \rightarrow (1)$

$$V_2 = I_L Z_L = -I_2 Z_L \rightarrow (2)$$

Sub (2) in (1)

$$I_2 = h_f I_1 - I_2 Z_L h_o$$

$$I_2 + I_2 Z_L h_o = h_f I_1$$

$$I_2 (1 + Z_L h_o) = h_f I_1 \Rightarrow \frac{I_2}{I_1} = \frac{h_f}{1 + Z_L h_o}$$

$$A_I = \frac{-I_2}{I_1} = \frac{-h_f}{1 + Z_L h_o}$$

	<u>CE</u>	<u>CB</u>	<u>CC</u>
A_I	$\frac{-h_{fe}}{1 + Z_L h_{oe}}$	$\frac{-h_{fb}}{1 + Z_L h_{ob}}$	$\frac{-h_{fc}}{1 + Z_L h_{oc}}$

2) Input Impedance z_i

In the circuit R_s is the signal source resistance. The impedance seen when looking in to the amplifier terminals (1, 1') is the amplifier input impedance z_i .

$$z_i = \frac{V_1}{I_1}$$

From figure $V_1 = h_i I_1 + h_{re} V_2$

$$\text{So } Z_i = \frac{h_i I_1 + h_{re} V_2}{I_1} = h_i + h_{re} \frac{V_2}{I_1} \rightarrow \textcircled{1}$$

$$V_2 = -I_2 Z_L = A_I I_1 Z_L \quad \left[\because A_I = \frac{-I_2}{I_1} \right]$$

$$\textcircled{1} \Rightarrow Z_i = h_i + h_{re} \frac{A_I I_1 Z_L}{I_1}$$

$$Z_i = h_i + h_{re} A_I Z_L$$

$$Z_i = h_i - h_{re} Z_L \frac{h_f}{1 + h_o Z_L} \quad \left[\because A_I = \frac{-h_f}{1 + h_o Z_L} \right]$$

$$Z_i = h_i - \frac{h_f h_{re}}{\frac{1}{Z_L} + h_o}$$

$$Z_i = h_i - \frac{h_f h_{re}}{Y_L + h_o} \quad \left[\because Y_L = \frac{1}{Z_L} \right]$$

$$Z_i \quad \begin{array}{c} \text{CE} \\ h_{ie} - \frac{h_{fe} h_{re}}{Y_L + h_{oe}} \end{array}$$

$$\begin{array}{c} \text{CB} \\ h_{ib} - \frac{h_{fb} h_{rb}}{Y_L + h_{ob}} \end{array}$$

$$\begin{array}{c} \text{CC} \\ h_{ic} - \frac{h_{fc} h_{rc}}{Y_L + h_{oc}} \end{array}$$

3) voltage gain (A_V):

The ratio of output voltage V_2 to input voltage gives the voltage gain of the transistor

$$A_V = \frac{V_2}{V_1}$$

$$\text{Substituting } V_2 = -I_2 Z_L = A_I I_1 Z_L$$

$$\Rightarrow A_V = \frac{A_I I_1 Z_L}{V_1} = \frac{A_I Z_L}{V_1 / I_1} = \frac{A_I Z_L}{Z_i}$$

$$A_V \quad \begin{array}{c} \text{CE} \\ \frac{A_I Z_L}{Z_i} \end{array}$$

$$\begin{array}{c} \text{CB} \\ \frac{A_I Z_L}{Z_i} \end{array}$$

$$\begin{array}{c} \text{CC} \\ \frac{A_I Z_L}{Z_i} \end{array}$$

4) output Admittance (Y_0) :

$$Y_0 = \frac{I_2}{V_2} \text{ with } V_S = 0 \text{ and } R_L = \infty$$

From the circuit $I_2 = h_f I_1 + h_o V_2$

$$\text{Dividing by } V_2, \quad \frac{I_2}{V_2} = h_f \frac{I_1}{V_2} + h_o \rightarrow \text{①}$$

with $V_S = 0$, by KVL in input circuit

$$R_S I_1 + h_i I_1 + h_{re} V_2 = 0$$

$$I_1 (R_S + h_i) + h_{re} V_2 = 0$$

$$\text{Hence } \frac{I_1}{V_2} = \frac{-h_{re}}{R_S + h_i}$$

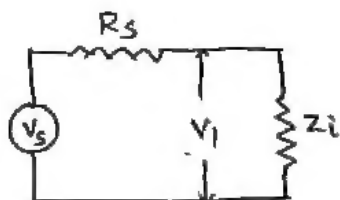
$$\text{Now Eq ①} \Rightarrow \frac{I_2}{V_2} = \frac{-h_f h_{re}}{R_S + h_i} + h_o$$

$$\Rightarrow Y_0 = h_o - \frac{h_f h_{re}}{R_S + h_i}$$

	CE	CB	CC
Y_0	$h_{oe} - \frac{h_{fe} h_{re}}{R_S + h_{ie}}$	$h_{ob} - \frac{h_{fb} h_{rb}}{R_S + h_{ib}}$	$h_{oc} - \frac{h_{fc} h_{rc}}{R_S + h_{ic}}$

5) Voltage gain (A_{VS}) (Including source) :

$$A_{VS} = \frac{V_2}{V_S} = \frac{V_2}{V_1} \frac{V_1}{V_S} \Rightarrow A_{VS} = A_V \frac{V_1}{V_S}$$



$$V_1 = \frac{V_S Z_i}{R_S + Z_i} \Rightarrow \frac{V_1}{V_S} = \frac{Z_i}{R_S + Z_i}$$

$$\text{Now } A_{VS} = \frac{A_V Z_i}{R_S + Z_i}$$

$$A_{VS} = \frac{A_I R_L}{Z_i} \times \frac{Z_L}{R_S + Z_L} = \frac{A_I R_L}{R_S + Z_L}$$

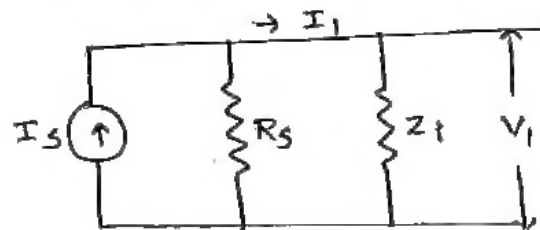
if $R_S = 0$ then $A_{VS} = \frac{A_I R_L}{Z_i} = A_V$.

6) Current Amplification (A_{IS})

$$A_{IS} = \frac{-I_2}{I_S} = \frac{-I_2}{I_1} \cdot \frac{I_1}{I_S} = A_I \frac{I_1}{I_S}$$

The modified input circuit using Norton's equivalent circuit for the source for the calculation of A_{IS}

$$A_{IS} = A_I \frac{R_S}{R_S + Z_i}$$



$$A_{VS} = \frac{A_{IS} Z_L}{R_S}$$

⇒ In CE configuration

current gain $A_I = \frac{-h_{fe}}{1 + h_{oe} Z_L} \quad [Z_L = R_L]$

Input Impedance $Z_i = h_{ie} - \frac{h_{fe} h_{ne}}{Y_L + h_{oe}} \quad [Y_L = \frac{1}{Z_L} = \frac{1}{R_L}]$

voltage gain $A_V = A_I \frac{Z_L}{Z_i}$

output Admittance $Y_o = h_{oe} - \frac{h_{fe} h_{ne}}{h_{ie} + R_S}$

⇒ In CB configuration

current gain $A_I = \frac{-h_{fb}}{1 + h_{ob} Z_L}$

Input Impedance $Z_i = h_{ib} - \frac{h_{fb} h_{nb}}{Y_L + h_{ob}}$

voltage gain $A_V = A_I \frac{Z_L}{Z_i}$

output Admittance $Y_o = h_{ob} - \frac{h_{fb} h_{nb}}{h_{ib} + R_S}$

⇒ in cc configuration

$$\text{Current gain } A_I = \frac{-h_{fc}}{1 + h_{oc} Z_L}$$

$$\text{Input Impedance } Z_i = h_{ic} - \frac{h_{fc} h_{nc}}{Y_L + h_{oc}}$$

$$\text{Voltage gain } A_V = \frac{A_I Z_L}{Z_i}$$

$$\text{Output Admittance } Y_o = h_{oc} - \frac{h_{fc} h_{nc}}{h_{ic} + R_s}$$

Conversion formulae for hybrid parameters

CB

$$h_{ib} = \frac{h_{ie}}{1 + h_{fe}}$$

$$h_{nb} = \frac{h_{ie} h_{oe}}{1 + h_{fe}} - h_{ne}$$

$$h_{fb} = \frac{-h_{fe}}{1 + h_{fe}}$$

$$h_{ob} = \frac{h_{oe}}{1 + h_{fe}}$$

CC

$$h_{ic} = h_{ie}$$

$$h_{nc} = 1$$

$$h_{fc} = -(1 + h_{fe})$$

$$h_{oc} = h_{oe}$$

1) characteristics of common emitter Amplifier

- 1) Current gain A_I is high for $R_L < 10k\Omega$
- 2) The voltage gain is high for normal values of Load resistance R_L
- 3) The input resistance R_i is medium
- 4) The output resistance R_o is moderately high

Applications of common emitter amplifier:

1. of the three configurations CE amplifier alone is capable of providing both voltage gain and current gain.
2. The output resistance R_o and input resistance R_i are moderately high
3. CE amplifier is widely used for Amplification purpose

2) characteristics of common Base Amplifier:

1. Current gain is less than unity and its magnitude decreases with the increase of load resistance R_L
2. voltage gain A_v is high for normal values of R_L
3. The input resistance R_i is the lowest of all the three configurations.
4. The output resistance R_o is the highest of all the three configurations.

Applications of common base Amplifier

The CB Amplifier is not commonly used for Amplification purpose. It is used for

- 1) Matching a very low impedance source.
- 2) As a non inverting amplifier with voltage gain exceeding unity
- 3) For driving a high impedance load
- 4) As a constant current source.

3) characteristics of common collector Amplifier

1. For low value of R_L ($< 10k\Omega$) the current gain A_i is high and almost equal to that of a CE amplifier

2. The voltage gain A_v is less than unity.
3. The input resistance is the highest of all the three configurations.
4. The output resistance is the lowest of all the three configurations.

Applications of common collector Amplifier:

1. The CC Amplifier is widely used as a buffer stage between a high impedance source and low impedance load. (CC Amplifier is called emitter follower)

Comparison of Transistor Amplifier Configurations.

The characteristics of three configurations are summarized in table below. Here the quantities A_i , A_v , R_i , R_o and A_p (Power gain) are calculated for $R_L = R_s = 3\text{ k}\Omega$

Quantity	CB	CC	CE
A_i	0.98	47.5	-46.5
A_v	131	0.989	-131
A_p	128.38	46.98	6091.5
R_i	22.6 Ω	144 $\text{k}\Omega$	1065 Ω
R_o	1.72 $\text{M}\Omega$	80.5 Ω	45.5 $\text{k}\Omega$

Simplified CE Hybrid Model (or) Approximate CE

Hybrid model (Approximate Analysis):

As the h parameters themselves vary widely for the same type of transistor, it is justified to make approximations and simplify the expressions for A_I , A_v , A_p , R_i and R_o .

The behaviour of the transistor circuit can be obtained by using the simplified hybrid model. The h -parameter equivalent circuit of the transistor in the CE configuration is shown in figure below.

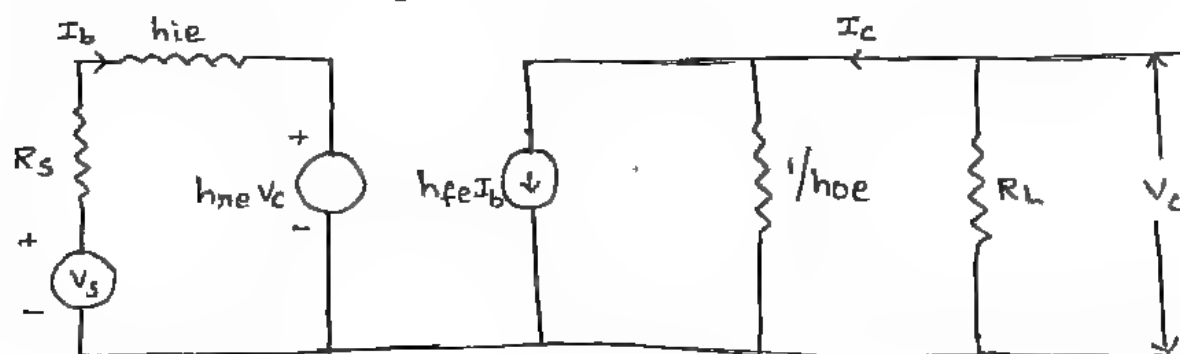


Fig: Exact CE Hybrid Model.

Here $\frac{1}{h_{oe}}$ is in parallel with R_L

The parallel combination of two unequal impedances is approximately equal to the lower value i.e. R_L . Hence if $\frac{1}{h_{oe}} \gg R_L$, then the term h_{oe} may be neglected

provided that $h_{oe}R_L \ll 1$

If h_{oe} is omitted, the collector current I_c is given by

$$I_c = h_{fe} I_b$$

under this condition the magnitude of voltage generated in the emitter circuit is

$$h_{ne} |V_c| = h_{ne} I_c R_L = h_{ne} h_{fe} I_b R_L$$

since $h_{ne} h_{fe} \approx 0.01$, this voltage may be neglected in comparison with the voltage drop across h_{ie} . ie $h_{ie} I_b$ provided that R_L is not too large. ie if the load resistance R_L is small it is possible to neglect the parameter h_{ne} and h_{oe} and the approximate equivalent circuit is obtained as shown in figure below.

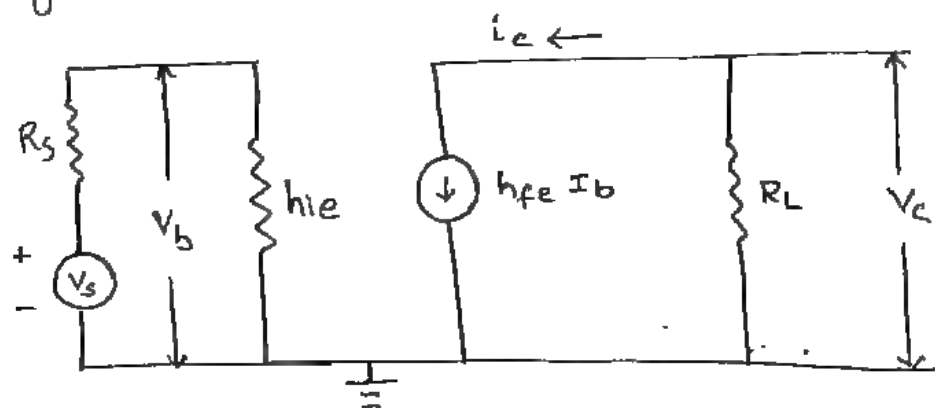


Fig: Approximate CE Hybrid model.

1) Current Gain (A_I):

The current gain for CE configuration is

$$A_I = \frac{-h_{fe}}{1 + h_{oe} R_L}, \quad \text{if } h_{oe} R_L < 0.1$$

$$A_I = -h_{fe}$$

2) Input Impedance (Z_i):

By exact analysis $Z_i = R_i = \frac{V_i}{I_i}$

$$V_1 = h_{ie} I_1 + h_{re} V_2$$

$$Z_i = \frac{h_{ie} I_1 + h_{re} V_2}{I_1} = h_{ie} + h_{re} \frac{V_2}{I_1}$$

$$V_2 = -I_2 Z_L = -I_2 R_L = A_I I_1 R_L \quad \left[\because A_I = \frac{-I_2}{I_1} \right]$$

$$\Rightarrow Z_i = h_{ie} + h_{re} \frac{A_I I_1 R_L}{I_1} \quad \left[\because V_2 = A_I I_1 R_L \right]$$

$$R_i = \left[h_{ie} + h_{re} A_I R_L \right]$$

$$R_i = h_{ie} \left[1 + \frac{h_{re} A_I R_L}{h_{ie}} \right]$$

$$R_i = h_{ie} \left[1 + \frac{h_{re} A_I R_L}{h_{ie}} \times \frac{h_{fe} h_{oe}}{h_{fe} h_{oe}} \right]$$

using the typical values for the h-parameters

$$\frac{h_{re} h_{fe}}{h_{ie} h_{oe}} \approx 0.5$$

$$\Rightarrow R_i = h_{ie} \left[1 + \frac{0.5 A_I R_L h_{oe}}{h_{fe}} \right]$$

we know that $A_I = \frac{-h_{fe}}{1 + h_{oe} R_L}$ if $h_{oe} R_L < 0.1$

then $A_I = -h_{fe}$

$$\Rightarrow R_i = h_{ie} \left[1 - \frac{0.5 h_{fe} R_L h_{oe}}{h_{fe}} \right]$$

$$\Rightarrow R_i = h_{ie} \left[1 - 0.5 h_{oe} R_L \right]$$

if $h_{oe} R_L < 0.1$

then $R_i = h_{ie}$ $\left[R_i = Z_i \right]$

voltage gain: $A_v = A_i \frac{R_L}{R_i} = \frac{-h_{fe} R_L}{h_{ie}}$

output Impedance:

It is the ratio of V_c to I_c with $V_s = 0$ and R_L excluded. The simplified circuit has infinite output impedance because with $V_s = 0$ and external voltage source applied at output, it is found that $I_b = 0$ and hence $I_c = 0$

$$R_o = \frac{V_c}{I_c} = \infty \quad [\because I_c = 0]$$

Approximate analysis of CE Amplifier

current gain $A_i = -h_{fe}$

Input resistance $R_i = h_{ie}$

Voltage gain $A_v = \frac{-h_{fe} R_L}{h_{ie}}$

output resistance $R_o = \infty$

Analysis of CC Amplifier using the approximate Model:

Figure shows the equivalent circuit of CC Amplifier using the approximate model with the collector grounded, input signal applied between base and ground and load connected between emitter and ground.

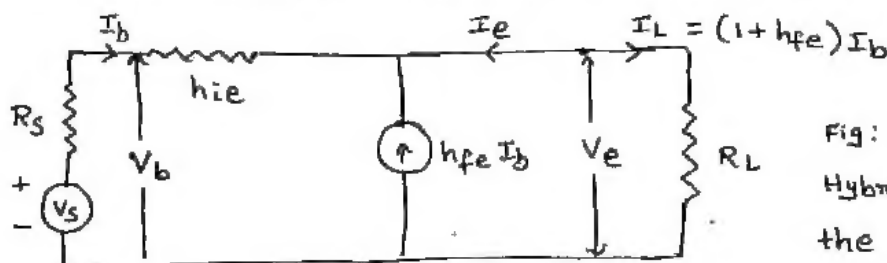


Fig: simplified Hybrid model for the CC circuit

1) current gain :-

$$A_I = \frac{I_L}{I_b} = \frac{(1+h_{fe}) I_b}{I_b} = (1+h_{fe})$$

2) Input resistance

$$V_b = I_b h_{ie} + (1+h_{fe}) I_b R_L$$

$$R_i = \frac{V_b}{I_b} = h_{ie} + (1+h_{fe}) R_L$$

3) Voltage gain

$$A_v = \frac{V_e}{V_b} = \frac{(1+h_{fe}) I_b R_L}{[h_{ie} I_b + (1+h_{fe}) I_b R_L]}$$

$$A_v = \frac{(1+h_{fe}) R_L}{h_{ie} + (1+h_{fe}) R_L} = \frac{h_{ie} + (1+h_{fe}) R_L - h_{ie}}{h_{ie} + (1+h_{fe}) R_L}$$

$$A_v = 1 - \frac{h_{ie}}{h_{ie} + (1+h_{fe}) R_L}$$

$$A_v = 1 - \frac{h_{ie}}{R_i} \quad \left[\because R_i = h_{ie} + (1+h_{fe}) R_L \right]$$

4) Output Impedance :-

$$\text{output admittance } (Y_o) = \frac{\text{short circuit current in o/p terminals}}{\text{open circuit voltage b/n o/p terminals}}$$

$$\begin{aligned} \text{short circuit current} &= (1+h_{fe}) I_b = (1+h_{fe}) \frac{V_s}{R_s + h_{ie}} \\ \text{in output terminals} & \end{aligned}$$

$$\begin{aligned} \text{open circuit voltage} &= V_s \\ \text{b/n output terminals} & \end{aligned}$$

$$\therefore Y_o = \frac{1+h_{fe}}{R_s + h_{ie}} \Rightarrow R_o = \frac{h_{ie} + R_s}{1+h_{fe}}$$

$$\text{output impedance including } R_L \text{ ie } R_o' = R_o \parallel R_L$$

Analysis of CB Amplifier using the approximate model

Figure shows the equivalent circuit of CB amplifier using the approximate model, with the base grounded, input signal is applied between emitter and base and load connected between collector and base

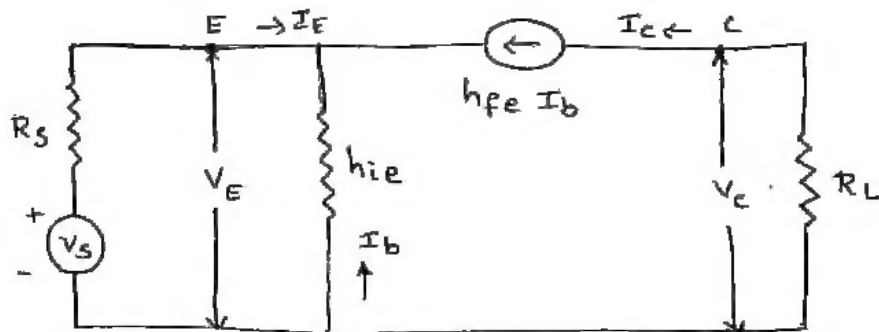


Fig.: Simplified Hybrid model for the CB circuit

1) current gain !

From the figure above $A_I = \frac{-I_c}{I_e} = \frac{-h_{fe} I_b}{I_e}$

$$I_e = -(I_b + I_c)$$

$$I_e = -(I_b + h_{fe} I_b) = -(1 + h_{fe}) I_b$$

$$\therefore A_I = \frac{-h_{fe} I_b}{-(1+h_{fe}) I_b} = \frac{h_{fe}}{1+h_{fe}} = -h_{fb}$$

2) Input Resistance:

Input Resistance $R_i = \frac{V_e}{I_e}$

From figure $V_e = -I_b h_{ie}$, $I_e = -(1+h_{fe}) I_b$

$$R_i = \frac{h_{ie}}{1 + h_{fe}} = h_{ib}$$

3) voltage gain:

$$A_v = \frac{V_c}{V_e}$$

$$V_c = -I_c R_L = -h_{fe} I_b R_L$$

$$V_e = -I_b h_{ie}$$

$$A_v = \frac{h_{fe} R_L}{h_{ie}}$$

output Impedance

$$R_o = \frac{V_c}{I_c} \quad \text{with} \quad V_s = 0, \quad R_L = \infty$$

With $V_s = 0$, $I_e = 0$ and $I_b = 0$ hence $I_c = 0$

$$\therefore R_o = \frac{V_c}{0} = \infty$$

Approximate Analysis of CB Amplifier

- 1) current gain $A_I = \frac{h_{fe}}{1+h_{fe}} = -h_{fb}$
- 2) Input Resistance $R_i = \frac{h_{ie}}{1+h_{fe}} = h_{ib}$
- 3) voltage gain $A_v = \frac{h_{fe} R_L}{h_{ie}}$
- 4) output resistance $R_o = \infty$

Approximate Analysis of CC Amplifier

- 1) current gain $A_I = (1+h_{fe})$
- 2) Input resistance $R_i = h_{ie} + (1+h_{fe}) R_L$
- 3) voltage gain $A_v = 1 - \frac{h_{ie}}{R_i}$
- 4) output Resistance $R_o = \frac{h_{ie} + R_s}{1+h_{fe}}$